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Ian Christensen and Ali Dib

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Ian Christensen[1] and Ali Dib[2]

[1]Monetary and Financial Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
ichristensen@bankofcanada.ca

[2]International Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
adib@bankofcanada.ca

The views expressed in this paper are those of the authors.
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Abstract

The authors estimate a sticky-price dynamic stochastic general-equilibrium model with a financial accelerator, à la Bernanke, Gertler, and Gilchrist (1999), to assess the importance of financial frictions in the amplification and propagation of the effects of transitory shocks. Structural parameters of two models, one with and one without a financial accelerator, are estimated using a maximum-likelihood procedure and post-1979 U.S. data. The estimation and simulation results provide some quantitative evidence in favour of the financial-accelerator model. The financial accelerator appears to play an important role in investment fluctuations, but its importance for output depends on the nature of the initial shock.

JEL classification: E32, E37, E44
Bank classification: Business fluctuations and cycles; Economic models; Econometric and statistical methods

Résumé

Les auteurs estiment un modèle d'équilibre général dynamique et stochastique où les prix sont rigides et qui intègre un mécanisme d'accélérateur financier, à la Bernanke, Gertler et Gilchrist (1999), afin d'évaluer l'importance des frictions financières dans l'amplification et la propagation des effets des chocs transitoires. Les paramètres structurels des deux modèles étudiés (dont l'un comporte un mécanisme d'accélérateur financier et l'autre pas) sont estimés au moyen de la méthode du maximum de vraisemblance à partir de données américaines remontant jusqu'à 1979. Les résultats des estimations et des simulations effectuées sont favorables, en termes quantitatifs, au modèle de l'accélérateur financier. Ce mécanisme semble jouer un rôle déterminant dans les fluctuations de l'investissement, mais son importance du point de vue de la production dépend de la nature du choc initial.

Classification JEL : E32, E37, E44
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Méthodes économétriques et statistiques

1. Introduction

Policy-makers, academics, and the business media often follow and discuss credit market conditions extensively. Newspaper stories highlighting the impending effects of "tight" or "easy" credit are common. The regular public communications of central banks analyze interest rate spreads or discuss recent trends in the growth of business lending. This discussion reflects a view that the ability of firms to obtain financing plays an active role in investment behaviour. Bernanke and Gertler (1989) show that the presence of asymmetric information in credit markets can give the balance sheet conditions of borrowers a role to play in the business cycle through their impact on the cost of external finance. The procyclical nature of net worth leads the wedge between the cost of external finance and internal funds, the external finance premium, to fall during booms and to rise during recessions. Bernanke, Gertler, and Gilchrist (1999, BGG hereafter), and others, including Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997), demonstrate that these financial frictions may significantly amplify the magnitude or persistence of fluctuations in economic activity. Despite this interest among researchers, mainstream macroeconomic models used for monetary policy analysis, such as the models used by Christiano, Eichenbaum, and Evans (2005), contain no role for financial frictions.

One reason for the omission of financial frictions from standard models is that there is little agreement about their importance for business cycle fluctuations. As a result, quantifying the importance of credit market frictions continues to be the subject of much research. To this end, we estimate a sticky-price dynamic stochastic general-equilibrium (DSGE) model similar to that of Ireland (2003), but with the addition of the financial friction described in BGG. We investigate whether this financial friction can improve the estimated model's ability to account for key features of the data, particularly those related to output and investment. We also assess the nature of the role it plays in the estimated model's dynamics.

Based on earlier work by Bernanke and Gertler (1989), BGG develop a model in which there is a link between the borrowing costs of firms and their net worth.[1] This link has come to be known as the "financial accelerator." In this model, en-

[1] An alternative approach is to introduce financial frictions by giving financial intermediaries an ability to change credit conditions without a change in borrower creditworthiness. Examples of these studies are Cook (1999), Cooper and Ejarque (2000), Atta-Mensah and Dib (2003), and Meh and Moran (2004).

trepreneurs, who borrow funds to undertake investment projects, face an external finance premium that rises when their leverage increases. A tightening in monetary policy, for example, reduces the return on capital, in part because the rental rate of capital falls and in part because of the drop in the value of that capital. The result is that the net worth of firms, which depends on the return to capital, declines. Declines in net worth increase firm leverage, leading to tighter financing conditions. This reduces the demand for capital, which reinforces the decline in its value. This mechanism is often called an "accelerator" effect, because declines in the net worth of firms raise the cost of financing, which has a feedback effect on net worth.

Carlstrom and Fuerst (1997) first demonstrated the quantitative importance of the Bernanke and Gertler (1989) mechanism, finding that it could produce a hump-shaped output response to shocks in an otherwise standard real business cycle model. The propagation brought about by the financial friction allowed the model to better match this key feature of the data, but it did not amplify the response of output. Using a sticky-price model calibrated to post-war U.S. data, BGG show that a different set-up for the financial-accelerator mechanism both amplifies the impact of shocks and provides a quantitatively important mechanism that propagates shocks at business cycle frequencies.[2]

In this paper, we develop and estimate a sticky-price DSGE model that includes the financial-accelerator mechanism proposed by Bernanke, Gertler, and Gilchrist (1999). The structural parameters of the model, including those related to the financial accelerator, are estimated econometrically using post-1979 U.S. macroeconomic data and a maximum-likelihood procedure with a Kalman filter. We also estimate a constrained version of the model in which the financial accelerator is turned off. Estimating these two versions of the model allows us to econometrically test for the presence of a financial-accelerator mechanism using the likelihood-ratio test. To evaluate the importance of the accelerator, we compare the impulse responses of key macroeconomic variables generated in models with and without the financial

[2]Subsequent work using the BGG model for other countries has provided similar results (see Hall 2001 for the United Kingdom and Fukunaga 2002 for Japan). A number of studies have used this financial-accelerator mechanism to account for macroeconomic developments at times of financial crisis. Cespedes, Chang, and Velasco (2004), Gertler, Gilchrist, and Natalucci (2003), Tovar (2003, 2004), and Elekdag, Justiniano, and Tchakarov (2005) consider the case of open economies in emerging markets. Christiano, Motto, and Rostagno (2004) use the financial accelerator in their analysis of the Great Depression in the United States.

accelerator.

We find that the estimate of the parameter related to the financial accelerator is statistically significant and larger than in many calibrated studies. The impulse-response functions show that introducing the financial accelerator greatly amplifies and propagates the effects of all transitory shocks on investment. Its importance for the amplification of output fluctuations varies, depending on the nature of the shock considered. The likelihood-ratio test rejects the basic sticky-price model without the financial accelerator in favour of the one that includes it.

These findings contrast with those reported by Meier and Muller (2005), who consider the role of the BGG-style financial accelerator in the monetary transmission mechanism. They estimate their model by matching impulse responses with the empirical impulse responses to a monetary policy shock from a vector autoregression. Their findings attribute an important role to capital adjustment costs, but only a marginal role to the accelerator in explaining the transmission of monetary policy shocks. As a result, Meier and Muller argue that little is lost if DSGE models do not incorporate financial-accelerator effects. We find that the accelerator mechanism plays an important role in the transmission of monetary policy shocks. In addition, as Meier and Muller acknowledge, their assessment is based solely on the accelerator's role in the transmission of monetary shocks. We find that the accelerator plays a role in explaining the response of macro variables to a variety of other shocks, particularly an investment-specific technology shock.

The model we develop is based on BGG (1999) and the estimated model of Ireland (2003). Ireland's model is based on a relatively standard New Keynesian model with sticky prices and capital that is very similar to the BGG set-up. This has the advantage that we can compare our results on the accelerator with BGG, and compare empirical models with that of Ireland (2003). Ireland also uses investment data in his estimation, which is important in our context, since we are most interested in the interaction of the price of capital, financing costs, and investment. Ireland's (2003) model has the advantage that it uses a general class of monetary policy rule that embeds a Taylor-type rule. This is useful because the behaviour of the monetary authorities has an impact on the quantitative importance of the financial accelerator. For example, BGG have noted that policy rules that stabilize output will also counteract, and may eliminate, the impact of the financial accelerator on

output or investment (see Fukunaga 2002 for an example).[3]

This paper is organized as follows. Section 2 describes the model. Section 3 describes the data and the econometric method used to estimate the models. Section 4 discusses the empirical results and section 5 offers some conclusions.

2. The Model

Our basic model is a closed-economy DSGE model similar to that of Ireland (2003). The key addition to this model is a financial-accelerator mechanism similar to that proposed by BGG. As a result, we assume that the economy is characterized by three types of rigidities: price stickiness, capital adjustment costs, and financial market frictions. We also assume that the economy is disturbed by five transitory shocks: technology, money demand, monetary policy, preference, and investment efficiency.

In this model there are three types of producers: entrepreneurs, capital producers, and retailers. Entrepreneurs produce intermediate goods. They borrow from a financial intermediary that converts household deposits into business financing for the purchase of capital. The presence of asymmetric information between entrepreneurs and lenders creates a financial friction that makes entrepreneurial demand for capital depend on their financial position. Capital producers build new capital and sell it to entrepreneurs. Changes in the supply of, or demand for, capital will lead the price of capital to fluctuate and further propagate the shocks. Retailers set nominal prices in a staggered fashion à la Calvo (1983).[4] This nominal rigidity gives monetary policy a role in this model. Our model differs from BGG in its characterization of monetary policy by a modified Taylor-type rule. We assume that the Federal Reserve adjusts short-term interest rates in response to inflation, output, and money-growth changes. In addition, we allow for the possibility of debt deflation and a utility function that is non-separable in consumption and real balances.

2.1 Households

The representative household derives utility from consumption, c_t; real money balances, M_t/p_t; and leisure, $1 - h_t$. Its preferences are described by the following

[3]See BGG (1999). The effects of the financial accelerator may, nonetheless, show up elsewhere, such as in the size of the monetary policy response required to dampen output fluctuations.

[4]Ireland (2003) introduces price stickiness using a quadratic price-adjustment cost function.

expected utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, M_t/p_t, h_t\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, M_t is holdings of nominal money balances, h_t is labour supply, and p_t is the consumer price level.[5] The single-period utility function is specified as:

$$u(\cdot) = \frac{\gamma z_t}{\gamma - 1} \log\left[c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma}\left(\frac{M_t}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right] + \eta \log\left(1 - h_t\right), \tag{2}$$

where γ and η are positive structural parameters that denote the constant elasticity of substitution between consumption and real balances, and the weight on leisure in the utility function, respectively. We interpret z_t as a taste (preference) shock, while b_t is interpreted as a money-demand shock. These shocks follow first-order autoregressive processes:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}, \tag{3}$$

and

$$\log(b_t) = (1 - \rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{4}$$

where $\rho_z, \rho_b \in (-1,1)$ are autoregressive coefficients, b is constant, and the serially uncorrelated shocks ε_{zt} and ε_{bt} are normally distributed with zero means and standard deviations σ_z and σ_b, respectively.

The representative household enters period t with d_{t-1} units of real deposits in the financial intermediary; nominal money balances, M_{t-1}; and nominal bonds, B_{t-1}. While deposits, d_t, at the financial intermediary pay the real interest rate, R_t, money balances, M_t, are money held outside of banks (cash) or savings instruments that bear low interest, such as chequing accounts.[6] During period t, the household chooses to consume, c_t; purchase new government bonds, B_t, for the price $1/R_t^n$ (where R_t^n is the riskless nominal interest rate); change nominal money balances,

[5] It has become standard to introduce money in the utility function to evaluate real balances in equilibrium, because money is dominated by other assets.

[6] The real return on bonds and deposits is the same in equilibrium. We introduce nominal (bonds) and real (deposits) assets to explicity derive the Fisher equation.

M_t; deposit funds at the financial intermediary, d_t; and work, h_t. The budget constraint is

$$c_t + \frac{d_t}{R_t} + \frac{M_t + B_t/R_t^n}{p_t} \leq \frac{W_t}{p_t} h_t + d_{t-1} + \frac{M_{t-1} + B_{t-1} + T_t + D_t}{p_t}, \tag{5}$$

where T_t denotes lump-sum transfers from the monetary authority and D_t refers to dividend payments received from retailer firms.

First-order conditions for the household optimization problem are:

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t, \tag{6}$$

$$\frac{z_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right). \tag{7}$$

$$\frac{\eta}{1 - h_t} = \lambda_t w_t, \tag{8}$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right]. \tag{9}$$

$$\frac{1}{R_t^n} = \beta E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1} \lambda_t} \right]. \tag{10}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint and $m_t = M_t/p_t$, $w_t = W_t/p_t$, $\pi_{t+1} = p_{t+1}/p_t$, are, respectively, real money balances, real wages, and the gross inflation rate.

2.2 Production sector

2.2.1 Entrepreneurs

The entrepreneurs' behaviour follows that proposed by Bernanke, Gertler, and Gilchrist (1999). Entrepreneurs manage firms that produce wholesale goods and borrow to finance the capital used in the production process. Entrepreneurs are risk neutral and have a finite expected horizon for planning purposes. The probability that an entrepreneur will survive until the next period is ν, so the expected lifetime horizon is $1/(1 - \nu)$. This assumption ensures that entrepreneurs' net worth (the firm equity) will never be enough to fully finance the new capital acquisition. In

essence, they issue debt contracts to finance their desired investment expenditures in excess of net worth.

At the end of each period, entrepreneurs purchase capital, k_{t+1}, that will be used in the next period at the price q_t. Thus, the cost of the purchased capital is $q_t k_{t+1}$. The capital acquisition is financed partly by their net worth, n_{t+1}, and by borrowing, $q_t k_{t+1} - n_{t+1}$, from a financial intermediary. This intermediary obtains its funds from household deposits and faces an opportunity cost of funds equal to the economy's nominal riskless rate of return, R_t^n.

The entrepreneurs' demand for capital depends on the expected marginal return and the expected marginal external financing cost at $t+1$, $E_t f_{t+1}$, which equals the real interest rate on external (borrowed) funds. Consequently, the optimal entrepreneurs' capital demand guarantees that

$$E_t f_{t+1} = E_t \left[\frac{r_{kt+1} + (1-\delta) q_{t+1}}{q_t} \right], \tag{11}$$

where δ is the capital depreciation rate, while the expected marginal return of capital is given by the right-side terms of (11), in which r_{kt+1} is the marginal productivity of capital at $t+1$ and $(1-\delta)q_{t+1}$ is the value of one unit of capital used in production in $t+1$.

BGG (1999) assume the existence of an agency problem that makes external finance more expensive than internal funds. The entrepreneurs costlessly observe their output, which is subject to a random outcome. The financial intermediaries incur an auditing cost to observe the output. After observing their project outcome, entrepreneurs decide whether to repay their debt or to default. If they default, the financial intermediaries audit the loan and recover the project outcome, less monitoring costs.

Accordingly, the marginal external financing cost is equal to a gross premium for external funds plus the gross real opportunity costs equivalent to the riskless interest rate. Thus, the demand for capital should satisfy the following optimality condition[7]:

$$E_t f_{t+1} = E_t \left[S(\cdot) R_t \right], \tag{12}$$

[7]For more details, see BGG (1999), who derive an optimal contract between entrepreneurs and financial intermediaries under an asymmetric information problem.

where $E_t R_t = E_t \left(R_t^n / \pi_{t+1} \right)$ is a riskless real interest rate,[8]

$$S(\cdot) = E_t S \left(\frac{n_{t+1}}{q_t k_{t+1}} \right), \tag{13}$$

with $S'(\cdot) < 0$ and $S(1) = 1$.

The gross external finance premium $S(\cdot)$ depends on the size of the borrower's equity stake in a project (or, alternatively, the borrower's leverage ratio). As $n_{t+1}/q_t k_{t+1}$ falls, the borrower relies on uncollateralized borrowing (higher leverage) to a larger extent to fund the project. Since this increases the incentive to misreport the outcome of the project, the loan becomes riskier and the cost of borrowing rises.[9]

From this relationship, we derive the log-linearized equation for the external finance premium:

$$\hat{f}_{t+1} - \hat{R}_t = -\psi \hat{n}_{t+1} + \psi \hat{k}_{t+1} + \psi \hat{q}_t, \tag{14}$$

where ψ represents the elasticity of the external finance premium with respect to a change in the leverage position of entrepreneurs.

Aggregate entrepreneurial net worth evolves according to

$$n_{t+1} = \nu v_t + (1 - \nu) g_t, \tag{15}$$

where v_t denotes the net worth of surviving entrepreneurs net of borrowing costs carried over from the previous period, $1 - \nu$ is the share of new entrepreneurs entering the economy, and g_t is the transfer or "seed money" that newly entering entrepreneurs receive from entrepreneurs who die and depart from the scene.[10] v_t is given by

$$v_t = \left[f_t q_{t-1} k_t - E_{t-1} f_t (q_{t-1} k_t - n_t) \right], \tag{16}$$

where f_t is the ex post real return on capital held in t, and $E_{t-1} f_t$ is the cost of borrowing (the interest rate in the loan contract signed in time $t - 1$). Earnings from operations in this period become next period's net worth. In our formulation,

[8]We derive this equation from (9) and (10), assuming that the covariance of λ_{t+1} and π_{t+1} equals zero.

[9]When the riskiness of loans increases, the agency costs rise and the lender's expected losses increase. A higher external finance premium paid by successful entrepreneurs offsets these higher losses and ensures that there is no change to the return on deposits for households.

[10]The parameter ν will affect the persistence of changes in net worth.

borrowers sign a debt contract that specifies a fixed nominal interest rate.[11] The loan repayment (in real terms) will then depend on the ex post real interest rate (see equation (C.17) in Appendix C). An unanticipated increase (decrease) in inflation will reduce (increase) the real cost of debt repayment.

To produce output y_t, the entrepreneurs use k_t units of capital and h_t units of labour following a constant-returns-to-scale technology:

$$y_t \leq k_t^{\alpha} \left(A_t h_t\right)^{1-\alpha}, \quad \alpha \in (0,1), \tag{17}$$

where A_t is a technology shock that is common to all entrepreneurs. The technology shock A_t is assumed to follow the autoregressive process

$$\log A_t = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{18}$$

where ρ_a (-1,1), $A > 0$, and ε_{At} is normally distributed with zero mean and standard deviation σ_A.

The entrepreneur maximizes profits by choosing k_t and h_t subject to the production function (17). The first-order conditions for this optimization problem are

$$r_{kt} = \alpha \frac{y_t}{k_t} \frac{\xi_t}{\lambda_t}, \tag{19}$$

$$w_t = (1-\alpha) \frac{y_t}{h_t} \frac{\xi_t}{\lambda_t}, \tag{20}$$

$$y_t = k_t^{\alpha} \left(A_t h_t\right)^{1-\alpha}, \tag{21}$$

where $\xi_t > 0$ is the Lagrangian multiplier associated with the technology function; ξ_t/λ_t is the real marginal cost, MC_t/p_t; w_t is the real wage; and r_t^k is the real rental rate on capital.[12]

2.2.2 Capital producers

Capital producers use a linear technology to produce capital goods, k_t, sold at the end of period t. They use a fraction of final goods purchased from retailers as investment goods, i_t, and the existing capital stock to produce new capital goods. The

[11] In BGG, the contract is specified in terms of the real interest rate.

[12] We assume that entrepreneurial consumption is small and it drops out of the model.

new capital goods replace depreciated capital and add to the capital stock. We assume that capital producers are subject to quadratic capital adjustment costs. Their optimization problem, in real terms, consists of choosing the quantity of investment to maximize profits, so that:

$$\max_{i_t} E_t \left[q_t i_t - i_t - \frac{\chi}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 k_t \right]. \tag{22}$$

Thus, the optimal condition is

$$E_t \left[q_t - 1 - \chi \left(\frac{i_t}{k_t} - \delta \right) \right] = 0, \tag{23}$$

which is the standard Tobin's Q equation that relates the price of capital to the marginal adjustment costs.

The quantity and price of capital are determined in the market for capital. The entrepreneurial demand curve for capital is determined by equation (11), and the supply of capital is given by equation (23). The intersection of these curves gives the quantity and price of capital. Capital adjustment costs slow down the response of investment to different shocks, which directly affects the price of capital.

Furthermore, the aggregate capital stock evolves according to

$$k_{t+1} = x_t i_t + (1 - \delta) k_t, \tag{24}$$

where δ is the depreciation rate and the disturbance x_t is a shock to the marginal efficiency of investment (as in Greenwood, Hercowitz, and Huffman 1988).[13] Since i_t is expressed in consumption units, x_t determines the amount of capital in efficiency units that can be purchased for one unit of consumption. The x_t shock follows the autoregressive process:

$$\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_{xt}, \tag{25}$$

where $\rho_x \ \epsilon(-1, 1)$ is an autoregressive coefficient, and ε_{xt} is normally distributed with mean zero and standard deviation σ_x.

[13]Greenwood, Hercowitz, and Krusell (2000) find that investment-specific technological progress is a source of about 30 per cent of output fluctuations. They point to the negative co-movement of the relative price of new capital and equipment investment as motivation for the use of this type of shock.

2.2.3 Retailers

Retailers purchase the wholesale goods at a price equal to nominal marginal costs, MC_t (the marginal cost in the entrepreneurs' sector),[14] and differentiate them at no cost.[15] They then sell these differentiated retail goods in a monopolistically competitive market. Following Calvo (1983), we assume that retailers cannot change their selling prices unless they receive a random signal. The constant probability of receiving such a signal is $(1-\phi)$. Thus, each retailer, j, sets the price, $\tilde{p}_t(j)$, that maximizes the expected profit for l periods.[16] The retailer's optimization problem is

$$\max_{\{p_t(j)\}} E_0 \left[\sum_{t=0}^{\infty} (\beta\phi)^l \lambda_{t+l} D_{t+l}(j)/p_{t+l} \right], \tag{26}$$

subject to[17]

$$y_{t+l}(j) = \left(\frac{\tilde{p}_t(j)}{p_{t+l}} \right)^{-\theta} y_{t+l}, \tag{27}$$

where the retailer's profit function is

$$D_{t+l}(j) = (\tilde{p}_t(j) - MC_{t+l})\, y_{t+l}(j). \tag{28}$$

The first-order condition is

$$\tilde{p}_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} MC_{t+l} y_{t+l}(j)/p_{t+l}}{E_t \sum_{l=0}^{\infty} (\beta\phi)^l \lambda_{t+l} y_{t+l}(j)/p_{t+l}}. \tag{29}$$

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[14] The entrepreneurs sell their output in a perfectly competitive market, so the price of their goods equals the marginal cost of production.

[15] The retail sector is used only to introduce nominal rigidity into this economy.

[16] Thus, l is the average length of time a price remains unchanged, $l = 1/(1-\phi)$.

[17] This demand function is derived from the definition of aggregate demand as the composite of individual final output (retail) goods and the corresponding price index in the monopolistic competition framework, as follows:

$$y_{t+l} = \left(\int_0^1 y_{t+l}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}},$$

$$p_{t+l} = \left(\int_0^1 p_{t+l}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}},$$

where $y_{t+l}(j)$ and $p_{t+l}(j)$ are the demand and price faced by each individual retailer, $j \in (0,1)$.

The aggregate price is

$$p_t^{1-\theta} = \phi p_{t-1}^{1-\theta} + (1-\phi)\bar{p}_t^{1-\theta}. \tag{30}$$

These equations lead to the following New Keynesian Phillips curve:

$$E_t\hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1-\beta\phi)(1-\phi)}{\phi}\hat{mc}_t, \tag{31}$$

where mc_t is real marginal cost, and variables with hats are log deviations from the steady-state values (such as $\hat{\pi}_t = \log(\pi_t/\pi)$).

2.3 Monetary authority

Following Ireland (2003), the central bank adjusts the nominal interest rate, R_t^n, in response to deviations of inflation ($\pi_t = p_t/p_{t-1}$), output (y_t), and the money-growth rate ($\mu_t = M_t/M_{t-1}$) from their steady-state values. Thus, the monetary policy rule evolves according to:

$$\log(R_t^n/R^n) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \varepsilon_{Rt}, \tag{32}$$

where R^n, π, y, and μ are the steady-state values of R_t^n, π_t, y_t, and μ_t, respectively, and ε_{Rt} is a monetary policy shock normally distributed with zero mean and standard deviation σ_R. The newly created money is transferred to households, so $T_t = M_t - M_{t-1}$.

We choose this policy rule to provide flexibility in the characterization of monetary policy. The policy coefficients, ϱ_π, ϱ_y, and ϱ_μ, are chosen by the monetary authority. In this case, a unique equilibrium exists as long as the sum of ϱ_π and ϱ_μ exceeds unity. Our modified Taylor-type rule embeds the standard Taylor (1993) rule (when $\varrho_\mu = 0$) where the monetary authority changes interest rates in response to inflation and output deviations.[18] If ϱ_μ is non-zero, monetary policy can be considered to influence a linear combination of the interest rate and money growth to achieve a target for inflation. Alternatively, the central bank may simply respond to

[18]Under the standard Taylor rule, money supply responds passively to the changes in the nominal interest rates. Thus, money stock is totally determined by the money demand. By reacting to money growth, the central bank is able to offset the negative effects of money-demand shocks on economic activity; see Dib (2002).

money growth because it wishes to insulate the economy from the effects of money-demand shocks or, more simply, because it is an indication of future inflation.

Since the parameters in this rule are estimated, we let the data decide upon the best characterization of monetary policy over the 1979 to 2004 period. Estimating this rule is important for our exercise, because allowing for a stronger output-stabilizing response of monetary policy may affect our conclusions regarding the importance of the financial accelerator. Ireland (2003) estimates this rule using the same sample and finds evidence that the money-growth term enters significantly, but that the coefficient on output does not. He also finds that his estimated model can generate the autocorrelation in interest rates observed in the data, despite the absence of an interest rate smoothing term.

2.4 Symmetric equilibrium

In the symmetric equilibrium, all entrepreneurs are identical, so they make the same decision. In this economy, the symmetric equilibrium consists of an allocation $\{y_t, c_t, m_t, i_t, h_t, k_t, n_t\}$ and a sequence of prices and co-state variables $\{w_t, r_{kt}, R_t^n, R_t, f_t, q_t, \lambda_t, mc_t\}$ that satisfy the optimality conditions of households, capital producers, entrepreneurs, and retailers; the money-supply rule; and the stochastic processes for preferences, money demand, productivity, investment, and monetary policy shocks (see Appendix A).

Taking a log-linear approximation of the equilibrium system around steady-state values, and using Blanchard and Kahn's (1980) procedure, yields a state-space solution of the form:

$$\hat{\mathbf{s}}_{t+1} = \Phi_1 \hat{\mathbf{s}}_t + \Phi_2 \varepsilon_{t+1}, \tag{33}$$

$$\hat{\mathbf{d}}_t = \Phi_3 \hat{\mathbf{s}}_t, \tag{34}$$

where the state variable vector, $\hat{\mathbf{s}}_t$, includes predetermined and exogenous variables; $\hat{\mathbf{d}}_t$ is the vector of control variables; and the vector ε_t contains the random innovations. The coefficient matrices, Φ_1, Φ_2, and Φ_3, have elements that depend on the structural parameters of the model. Therefore, the state-space solution, (33) and (34), is used to estimate and simulate the model.

3. Data and Estimation Strategy

As in previous studies that estimate DSGE models using a maximum-likelihood procedure, some parameters are set prior to estimation because the data used in the estimations contain little information about them. Thus, the parameter η, denoting the weight on leisure in the utility function, is set equal to 1.315, so that the household spends around 33 per cent of its time in market activities. The degree of retailers' monopoly power, θ, is set equal to 6, which implies a gross steady-state price markup of 1.20, a common value used in the literature. The depreciation rate, δ, is assigned the commonly used values of 0.025. The constant associated with money demand, b, is set to 0.07, to ensure that the steady-state ratio of real balances to consumption is close to its historical value.

BGG solve a financial contract that maximizes the payoff to the entrepreneur, subject to the lender earning the required rate of return. BGG show that—given parameter values associated with the cost of monitoring the borrower, characteristics of the distribution of entrepreneurial returns, and the expected life span of firms—their contract implies a steady-state external finance premium and leverage ratio that are close to long-run historical averages observed in the data.[19] The underlying parameter values determine the elasticity of the external finance premium with respect to firm leverage (ψ, see equation (14)).

In our empirical model, we abstract from the parameters that underpin the financial contract. We calibrate the steady-state interest rate on external funds equal to the average of the business prime loan rate over our sample (this gives a gross external finance premium, $S(\cdot)$, of about 1.03, or 3.0 per cent annualized and on a net basis). We set the steady-state capital-to-asset ratio equal to 2. This implies a firm leverage ratio, defined as the ratio of debt to assets, of 0.5. The probability that an entrepreneur will survive for the next period, ν, is set to 0.9728, as in BGG (1999), implying that the expected working life of an entrepreneur is 36 years.

We set values for the steady-state external finance premium and the leverage ratio to historical averages of the same data that BGG try to match. Instead of fixing the value of the elasticity (ψ) based on this information, we estimate it using

[19] For details, see Appendix A of BGG (1999) or the appendix in Gertler, Gilchrist, and Natalucci (2003).

aggregate investment and the other macro series in our data set. Our objective is to determine whether financing constraints that depend on firm net worth can improve the fit of our model and allow it to match some stylized facts about investment.

The remaining non-calibrated parameters are estimated using a maximum-likelihood procedure with a Kalman filter. This method applies a Kalman filter to a model's state-space form to generate series of innovations used to evaluate the likelihood function for the sample. Because the solution is a state-space econometric model, driven by five innovations in ε_t, the structural parameters embedded in Φ_1, Φ_2, and Φ_3 can be estimated by a maximum-likelihood procedure using data for five series, in this case y_t, i_t, π_t, R_t^n, and m_t.[20]

Using quarterly U.S. data from 1979Q3 through 2004Q3, we estimate two versions of the model.[21] The first is a model with a financial accelerator (the FA model). The second is the same model with the dynamic effects of the financial accelerator turned off. In this second model, which we call the Estimated No-FA model, the parameter that captures the elasticity of the external finance premium with respect to firm leverage, ψ, is constrained to equal zero.[22] See the linearized equations in Appendix C for more details.

In the U.S. data, output is measured by real GDP excluding government expenditures, since there is no government spending in the model.[23] Since the financial friction in our model exerts influence directly on investment behaviour, we use investment data in the estimation. In addition, Ireland (2003) argues that investment data are required because it is insufficient to use only output data to identify the parameter of the capital adjustment cost. Investment is measured by real gross private domestic investment. Real money balances are measured by dividing the base money stock, M0, by the GDP deflator.[24] These three series are expressed in

[20]This method is described in Hamilton (1994, chapter 13).

[21]This period corresponds to the Volcker-Greenspan era at the Federal Reserve, which is often characterized as a period of relatively constant monetary policy. We can thus avoid the indeterminacy problems often found for models estimated with pre-1979 data.

[22]Both models are estimated with the same steady-state risk premium on external funds. In the estimated No-FA model, however, the risk premium is constrained to equal its steady-state value, rather than fluctuate with changes in firm net worth over the cycle.

[23]To construct the output series used in the estimation, we subtract government spending from the aggregate data on U.S. nominal GDP.

[24]We also conducted the estimation exercise with M1 as an alternative measure of money and found similar qualitative results.

per capita terms using the civilian population aged 16 and over. The inflation rate is measured by changes in the GDP implicit price deflator, while the short-term nominal interest rate is measured by the rate on three-month treasury bills. All the series are HP-filtered before the estimation, including inflation and interest rates, because they exhibit a small downward trend over the post-1979 sample.[25]

4. Empirical Results

4.1 Parameter estimates

Table 1 reports the maximum-likelihood estimates and standard errors of the FA and Estimated No-FA model's structural parameters for the period 1979Q3 to 2004Q3.

The estimate of the parameter ψ, the elasticity of the external finance premium with respect to firm leverage, is statistically significant and equal to 0.092. This estimate is higher than values usually used to calibrate this parameter in models with a financial accelerator. For example, Bernanke and Gertler (2000) set ψ to 0.05, about half of our estimated value. Meier and Muller (2005) report an estimated value of 0.067 for this parameter, but the estimate is not statistically significant.

The difference in parameter estimates associated with capital adjustment costs and the monetary policy rule across the two models also suggests that the accelerator mechanism is helping the models to account for fluctuations in investment. The capital adjustment cost parameter, χ, is 1.43 in the FA model, more than double the 0.64 estimated in the Estimated No-FA model. These estimates are considerably higher than the 0.25 value for the adjustment cost parameter used by BGG. Meier and Muller report an estimate of 0.65, below that in our FA model. Using a similar econometric methodology, however, Ireland (2001, 2003) finds estimates of the adjustment cost parameter that are much larger.[26] Capital adjustment costs have an important interaction with the financial-accelerator mechanism. If capital adjustment costs are high, the price of capital will respond to shocks to a greater extent. The price of capital has a direct effect on the net worth of firms (through capital gains and losses) and therefore on the cost of external financing. The higher

[25] In future work, we will consider the robustness of our results to alternative filtering procedures, such as linearizing around a linear deterministic trend or using a common stochastic trend.

[26] The estimated value for χ in Ireland (2003), in the post-1979 sample, is 32.1 in the sticky-price model, while it is 17.4 in the flexible-price model.

capital adjustment costs in the FA model suggest that the FA mechanism may be helping to generate investment volatility.

In both models, estimates of the policy rule parameters indicate that, since 1979, the Fed has responded much more strongly to inflation deviations than to output or money-growth fluctuations. This is particularly true for the FA model, which suggests a more aggressive response of monetary policy to inflation deviations than in the estimated No-FA model. The estimate of ϱ_π, the coefficient that measures the response of monetary policy to inflation deviations, is 1.94 in the FA model, but much less, 0.91, in the estimated No-FA model. The estimates of ϱ_y are small, but statistically significant, and take the expected sign in both models. The estimated value of ϱ_μ, the weight on money-growth deviations, is 0.41 in the FA model, but a much smaller 0.15 in the estimated No-FA model. The estimates of all the monetary policy rule parameters are statistically different from zero.

The larger estimated coefficients for the monetary policy rule in the model with a financial accelerator are not surprising. The presence of the financial accelerator leads to an amplification and propagation of the impacts of the shocks on output, inflation, and money growth. Thus, the monetary authority needs to respond more aggressively to changes in these variables to control inflation than it would if there were no financial accelerator.

The estimate of γ implies that money demand has an interest elasticity of -0.026, which is very close to the values estimated in Ireland (2003) for the post-1979 period. The estimate of the capital share in the production function, α, is close to 0.33, the value often assumed in the literature. The estimate of ϕ, the probability that prices remain unchanged for the next period, is about 0.5 in both models. This indicates that firms set prices for about two quarters, on average.[27] Thus, prices are quite flexible compared with other estimated DSGE models that have Calvo pricing.

We next examine the estimated shock processes. The investment-efficiency shock stands out in both models as having the largest volatility, but less so in the FA model. We therefore find that large investment shocks are important for the empirical model to explain the co-movement of investment and the other variables in the post-1979 data. Ireland's (2003) findings are similar, and he argues that large investment shocks are required for the model to explain the investment boom of the 1990s.

[27]Prices are somewhat stickier in BGG, with $\phi = 0.75$, implying an average period of four quarters between price adjustments.

There is a notable difference in the estimated persistence of the shocks across models. The estimates for the No-FA model show that the investment efficiency and preference shocks are highly persistent, while the estimates for the FA model are less so. In fact, all of the shocks are less persistent in the FA model, possibly because of the added propagation effects from net worth that the accelerator mechanism brings to the model.

Do the dynamic effects associated with fluctuations in net worth and the risk premium allow the FA model to better capture the co-movement in the data? We use the likelihood-ratio test to test the restriction imposed by the estimated No-FA model ($\psi = 0$) against the model with the financial accelerator (FA model). Let L^u and L^c denote the maximum values of the log-likelihood function for the unconstrained (FA) and constrained (estimated No-FA) models, respectively. The likelihood-ratio statistic $-2(L^c - L^u)$ has a chi-square distribution with one degree of freedom under the null hypothesis that the No-FA is valid. The value of L^u is 1896.8 and that of L^c is 1871.4, giving a test statistic of 50.8. The 1 per cent critical value for a $\chi^2(1)$ is 6.64. Therefore, the likelihood-ratio test easily rejects the restriction of the estimated No-FA model in favour of the model that includes a financial accelerator. The introduction of the accelerator mechanism improves the model's ability to capture the co-movement in the data.[28]

4.2 Impulse responses

We compare the responses of various macroeconomic variables with the five different shocks when the financial accelerator is present and when it is not. Figures 1 to 5, respectively, show the impulse responses to a 1 per cent shock to the short-term nominal interest rate (tightening of monetary policy), technology (increase in A_t), money demand (increase in b_t), preferences for consumption (increase in z_t affecting the marginal utility of consumption), and the efficiency of investment (increase in x_t). Each variable's response is expressed as the percentage deviation from its steady-state level, with the exception of rate variables, which are in percentage points (e.g., a 0.1 increase in $\hat{R}_t^n$ is an increase of 10 basis points).

[28]This is not an empirical test for the existence of a financial friction: one must exist in both models, because the steady-state cost of external funds exceeds the risk-free rate. This is a test of the extent to which such a friction improves the model's ability to account for the dynamics of macrovariables observed in the data.

In Figures 1 to 5, the impulse responses generated in the estimated FA model are shown in red. The dashed lines (in blue) show impulse responses when the dynamic effects of the financial accelerator are not present. They are the impulse responses generated by setting ψ equal to 0, but keeping all of the other parameter estimates from the FA model. We call this the No-FA model. The difference between the red and blue lines should indicate the impact of the accelerator mechanism on a given variable after a particular shock. Since the likelihood-ratio test rejects the estimated model in which ψ is constrained to equal zero, its impulse responses are not shown.

Figure 1 shows that the presence of a financial accelerator both amplifies and propagates the impact of a positive 1 per cent monetary policy shock on real variables, particularly for investment. Despite the fact that the shock lasts for only one period, deviations of investment, output, and hours are long-lived.[29] The basic mechanism of the financial accelerator is evident in the impulse responses. After a tightening in monetary policy, net worth falls, because of the declining return to capital and the higher real interest costs associated with existing debt (the debt-deflation effect). The external finance premium rises, reflecting the increase in firm leverage. The higher funding cost of purchasing new capital depresses the demand for it, and the expected price of capital persists below its steady-state value. These impulse responses show considerably more amplification of the response of investment than reported in Meier and Muller (2005), reflecting, in part, the higher estimated values that we find for ψ and χ.

Figure 2 shows that, following a 1 per cent positive technology shock, there is an important amplification of investment, but no amplification of the output response when the financial accelerator is present. The impact on output, investment, and hours lingers in the FA model responses. The technology shock increases the return to capital, pushing up net worth. The small decline in inflation that results from the shock increases the real cost of repaying existing debt, dampening slightly the rise of net worth. The positive impact on net worth from the higher return to capital dominates, due in part to the endogenous policy response that reduces the disinflationary impact, and net worth rises. Higher net worth decreases the external finance premium and increases the demand for capital. The response of investment to the shock is much larger when the FA is present. As is often found in sticky-

[29]The persistence of the decrease in output following a monetary policy shock is due to the persistence of the investment response.

price models, hours worked declines after the technology shock, since the wealth effect from higher marginal product of labour outweighs the substitution effect. The decline in hours worked, however, is not very different in the FA and No-FA cases.

Figure 3 shows the impulse responses to a positive 1 per cent money-demand shock. As the demand for real balances rises, consumption and savings falls, depressing output and investment. In addition, with less output being produced, but more liquidity expected in the economy, inflation rises. The monetary authority responds with higher interest rates and an increased supply of money, since the interest elasticity of money demand is small. In the FA model, the initial drop in the return to capital has a larger impact on output and investment, owing to the accelerator effects.

Figure 4 shows the impulse responses to a positive 1 per cent shock to the marginal utility of consumption and real balances. The presence of a financial accelerator dampens the impact of the shock slightly from the No-FA case, due to its influence on investment, which declines more sharply when the accelerator is present (consumption is almost identical in the two cases). The preference shock initially raises the marginal utility of consumption and therefore the opportunity cost of holding deposits (savings). As households divert deposits towards consumption, the return on deposits (the risk-free real interest rate) rises. In the accelerator model, the rise in this interest rate has a larger effect on investment, due to its impact on firms' net worth.

Figure 5 shows the impulse responses to an investment-efficiency shock, which enables us to show how the investment shock interacts with the financial-accelerator mechanism in this model. The investment shock is a persistent positive shock to the marginal efficiency with which investment goods are turned into capital. Impulse responses from the FA model show that, after such a shock, investment drops sharply but the capital stock increases: because of the higher marginal efficiency of investment, there are more effective units of capital, despite a decline in the amount of investment goods. Investment falls because the future marginal product of capital declines and capital adjustment costs increase as the capital stock rises. In the FA model, the decline in investment is more pronounced.[30] The rise in the supply of

[30]The instantaneous negative responses of output, hours, and investment to the investment shock in the No-FA model are explained by the high persistence of the investment-efficiency shock. If

capital reduces its price. The replacement cost of existing capital falls, lowering the return on capital and hence net worth. The resulting rise in the external finance premium raises the cost of funding investment purchases even higher. The fact that a positive productivity shock to investment causes an increase in the risk premium may be particular to the form of capital adjustment costs in the model.

As in previous studies, the FA amplifies and propagates the impact of the shocks on investment. The importance of the FA for output fluctuations, however, depends on the type of shock. For the monetary policy, money-demand, and investment-efficiency shocks, the initial impact on output is double (or more) when the FA is present. The FA, however, has no impact on the initial response of output after a technology shock, although the effects are more persistent. In the case of the shock to the marginal utility of consumption, output actually responds less when the FA is present.

4.3 Volatility and autocorrelation

To assess the contribution of the accelerator mechanism in our estimated model, we consider the model-implied volatilities and autocorrelations of the main variables of interest. Table 2 reports the volatilities of output, investment, money growth, interest rates, and inflation from the filtered data, and for simulated versions of the FA model with the accelerator active (FA) and with it turned off (No-FA).[31] The standard deviations are expressed in percentage terms. In the data, investment is about 5 times as volatile as output: the standard deviation of output is 1.04 and investment is 5.6. Money growth has a standard deviation of 0.85 per cent. The short-term nominal interest rate and inflation are less volatile; their standard deviations are 0.31 per cent and 0.21 per cent, respectively.

The simulation results show that, in the model where the accelerator is active, output volatility is close to that in the data. The model in which the accelerator is inactive overpredicts output volatility, however, a feature common in sticky-price models.

Both of the models overpredict the volatility of investment, but not the ratio of investment volatility to output volatility. Investment is almost 9 times as volatile as

we lower the persistence of this shock closer to the estimate in Ireland (2003), output, hours, and investment respond positively to the shock.

[31] In the data, all series are HP-filtered before calculating their standard deviations.

output, in the FA model, compared with about 5 times in the data. In the model with the FA inactive, investment is not even twice as volatile as output.

The FA model that contains an extra friction meant to amplify and propagate shocks shows less output volatility. At the bottom of Table 2, we report the volatility of output and investment with the investment shock shut off, to gain insight into its contribution to these findings. Investment volatility in the FA model becomes much larger than in the No-FA model, suggesting that the accelerator is amplifying the effects of other shocks on investment.

The FA and No-FA models are relatively successful at replicating the volatility of money growth, but they overpredict the volatility of nominal interest rates. Both models also increase the volatility of inflation by greater than the amount shown in the data.

Figure 6 plots the autocorrelation functions for output, investment, nominal interest rates, inflation, and real balances generated by our models and in the data. The model with the active FA mechanism does a better job at matching the autocorrelations shown in the data. It does a good job of matching the autocorrelation in inflation and the nominal interest rate within a four-quarter horizon. Output and investment in the FA model, however, are still much more highly autocorrelated than in the data. Nonetheless, the presence of the financial-accelerator mechanism greatly reduces the autocorrelation in output and investment. Both models generate too little autocorrelation in real balances relative to the data. Output is more persistent in the estimated No-FA model, because the estimated autoregressive coefficients of all the shocks are larger than in the FA model.

4.4 Variance decompositions

We next consider the forecast-error variance decompositions for output, investment, money growth, nominal interest rates, and inflation from the FA model with and without the active financial accelerator. Tables 3 and 4 show the forecast-error variance decompositions of the variables attributed to each of the five shocks for one- and ten-quarter-ahead horizons, respectively. In the financial-accelerator model, technology, preference, and investment-efficiency shocks account for the bulk of output fluctuations. Of these three shocks, only investment efficiency has important differences in the initial response of output when the accelerator mechanism is present. The amplification of this shock is therefore the main reason for the increase in the

forecast-error variance at short horizons. When the accelerator is turned off, preference shocks alone account for about half of the output fluctuations.[32] Without the accelerator, investment-efficiency shocks contribute to output fluctuations only at longer horizons.

Monetary policy shocks account for a small fraction of the variance in output and investment in either model, as Christiano, Eichenbaum, and Evans (2005) find. However, monetary policy shocks account for a larger share of the one-quarter-ahead output variance when the accelerator is active. Policy shocks also have effects on output fluctuatons at longer horizons in the presence of the accelerator. For example, monetary policy shocks account for over 5.2 per cent of ten-quarter-ahead forecast variance error of output in the FA model, but less than 1 per cent in the absence of the accelerator.

In both models, most of the fluctuation in investment is explained by investment-efficiency shocks, particularly in the presence of the financial accelerator, where investment shocks account for 96 per cent of the variance. This suggests that the models on their own, even with the financial-accelerator mechanism, require large shocks to account for the co-movement of the investment data with data on output, inflation, interest rates, and money growth.

5. Conclusion

In this paper, we estimate a standard sticky-price model with the addition of a financial friction along the lines of Bernanke, Gertler, and Gilchrist's (1999) financial accelerator. Our objectives are to determine whether the financial-accelerator mechanism can improve the sticky-price model's ability to fit post-1979 U.S. data, and to assess the nature of the mechanism's role in the estimated model's dynamics.

Using a maximum-likelihood procedure with a Kalman filter, we estimate two versions of the model: one with and one without the financial accelerator. The estimated value of a key parameter in the accelerator mechanism, the elasticity of the external finance premium with respect to firm leverage, is statistically significant and higher than values found in other empirical work or typical calibrations. A likelihood-ratio test finds an improvement in the model's fit with the data when

[32]This finding is different from that of Ireland (2003), where most of the output fluctuations are attributed to investment shocks.

the financial accelerator is active. In our empirical model, the financial accelerator amplifies and propagates the investment response to all of the transitory shocks considered. Its importance for the amplification of output responses varies, depending on the nature of the shock considered. Overall, the presence of the financial accelerator increases the variance of output at short horizons. We find that monetary policy shocks play a small role in output fluctuations in all of the models considered. However, the presence of the accelerator extends the horizon over which monetary policy shocks play a role in output fluctuations.

While these results are encouraging, the empirical model still requires large and persistent investment-efficiency shocks to fit the data. This suggests that one avenue for future model development should explore alternative formulations for the elements of the model related to investment. In addition, since both the accelerator mechanism and the investment-efficiency shock alter the relative price of consumption to investment, it will be important to test the robustness of these results in models where there is no investment-efficiency shock. Another useful extension to the model would be to explore other utility functions that make households less willing or able to substitute consumption intertemporally, since this could affect the impact of the accelerator on aggregate output. This might be achieved by households themselves facing a financial friction. Finally, future work should consider whether aggregate financial data can be used in the estimation to make a stronger link between our findings and firm financing.

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Table 1: Maximum-Likelihood Estimates: 1979Q3 to 2004Q3

	FA model		No-FA model	
Parameters	Estimates	Std. errors	Estimates	Std. errors
ψ	0.0922	0.0102	-	-
α	0.3412	0.0382	0.3234	0.0390
γ	0.0267	0.0048	0.0195	0.0034
χ	1.4264	0.5809	0.6428	0.3719
ϕ	0.4939	0.0091	0.4816	0.0731
ϱ_π	1.9440	0.2616	0.9141	0.0425
ϱ_y	0.0836	0.0256	0.0466	0.0188
ϱ_μ	0.4157	0.0502	0.1518	0.0320
σ_R	0.0050	0.0007	0.0033	0.0003
ρ_A	0.7599	0.0623	0.8502	0.0869
σ_A	0.0060	0.0008	0.0033	0.0007
ρ_b	0.6177	0.0531	0.8351	0.0623
σ_b	0.0101	0.0004	0.0085	0.0008
ρ_z	0.6684	0.0597	0.9565	0.0148
σ_z	0.0086	0.0008	0.0137	0.0017
ρ_x	0.8877	0.0364	0.9798	0.0251
σ_x	0.0841	0.0147	0.1207	0.0318
LL	1896.8		1871.4	

Table 2: Standard Deviations: Data and Models (in per cent)

Variables	Data	FA model	FA model, $\psi = 0$
y_t	1.55	1.22	4.85
i_t	6.54	10.78	8.54
μ_t	0.85	0.87	0.91
R_t^n	0.31	0.42	0.43
π_t	0.21	0.28	0.39
Excluding investment shock			
y_t	1.04	0.99	0.74
i_t	5.61	2.47	0.62

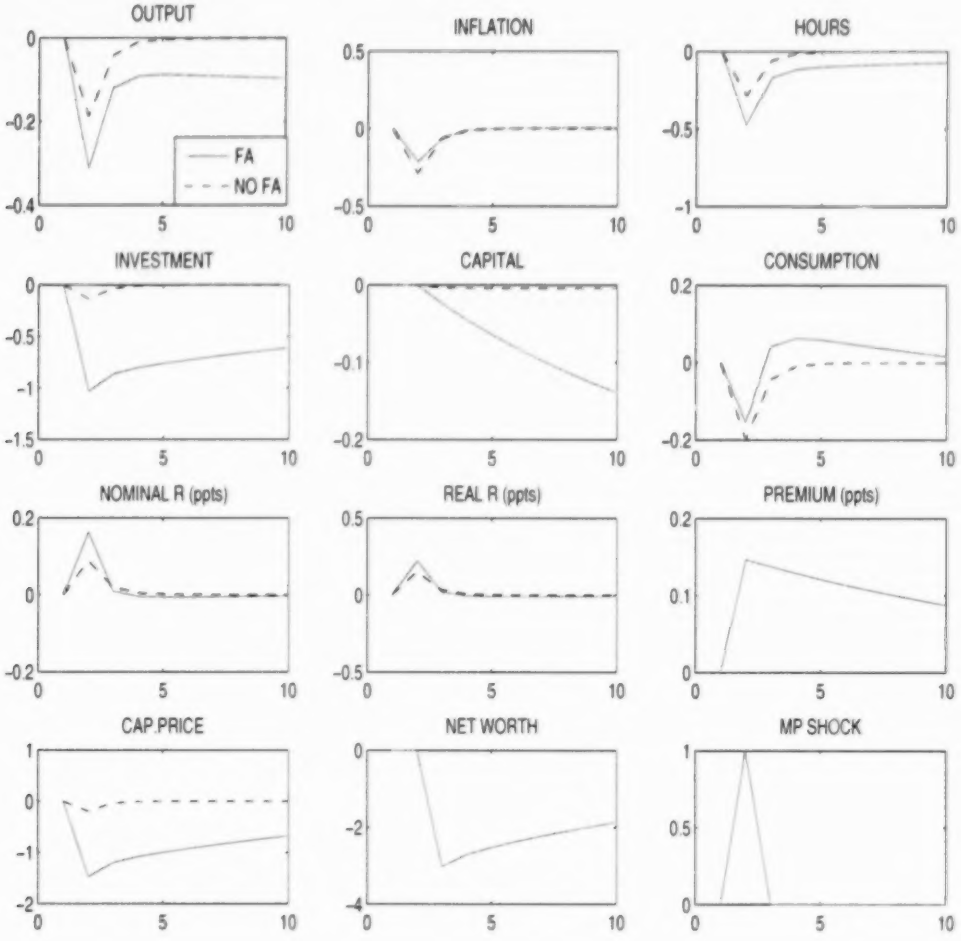
Table 3: One-Quarter-Ahead Forecast-Error Variance Decompositions

		Percentage owing to				
Variable	Variance	Technology	Money demand	Policy	Preference	Investment
Financial-accelerator model						
y_t	0.0036	26.4	4.4	6.7	26.3	36.1
i_t	0.1724	1.5	0.1	1.5	0.9	96.0
μ_t	0.0055	16.9	2.0	25.4	11.9	43.8
R_t^n	0.0009	0.1	50.1	7.2	42.4	0.2
π_t	0.0004	11.4	13.9	27.1	32.7	14.8
Financial-accelerator model ($\psi = 0$)						
y_t	0.0023	40.6	2.3	3.9	52.5	0.7
i_t	0.0054	7.9	0.4	0.8	8.8	82.0
μ_t	0.0071	46.3	42.5	9.6	1.2	0.5
R_t^n	0.0004	10.8	25.9	4.5	54.7	4.1
π_t	0.0005	40.5	5.1	42.9	9.7	1.8

Table 4: Ten-Quarter-Ahead Forecast-Error Variance Decompositions

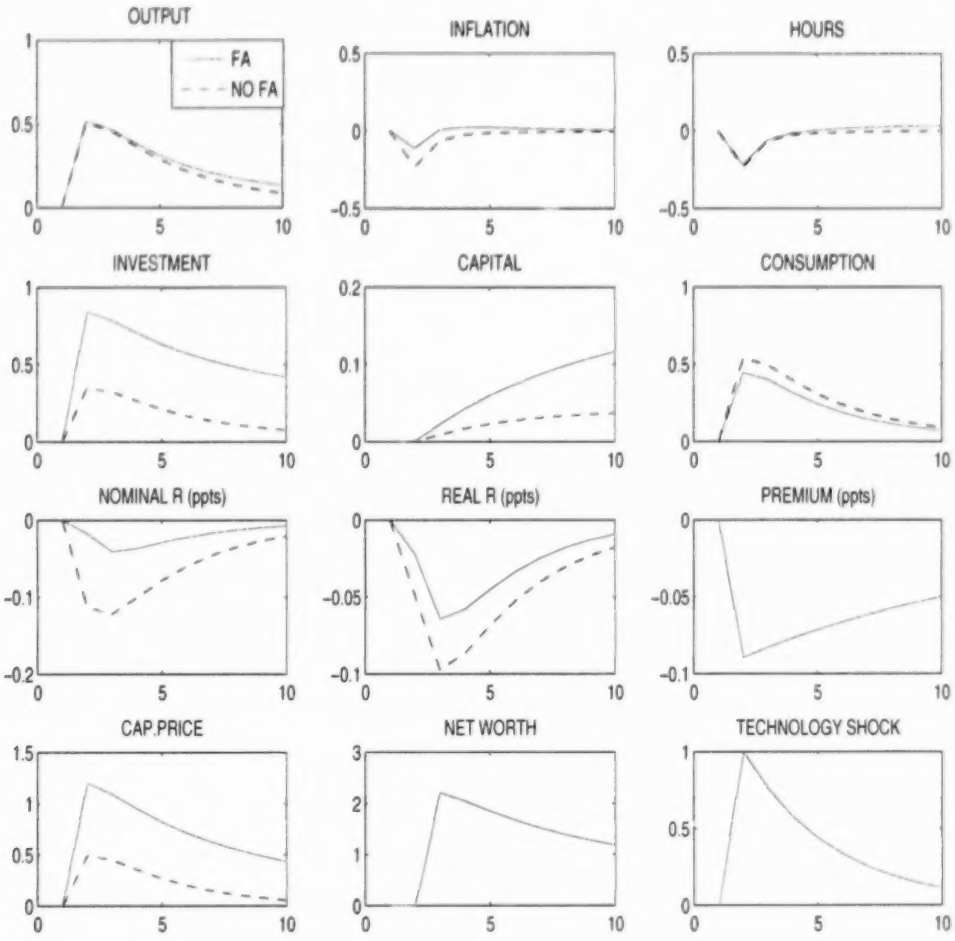
Variable	Variance	Percentage owing to				
		Technology	Money demand	Policy	Preference	Investment
		Financial-accelerator model				
y_t	0.0087	38.4	2.2	5.2	17.6	36.6
i_t	0.9072	1.4	0.1	1.5	0.5	96.5
μ_t	0.0399	9.5	1.6	3.6	3.4	81.9
R_t^n	0.0016	1.2	42.1	4.2	45.8	6.6
π_t	0.0007	7.2	33.9	16.6	27.8	14.5
		Financial-accelerator model ($\psi = 0$)				
y_t	0.0111	26.2	0.5	0.8	18.3	54.2
i_t	0.0165	9.0	0.1	0.3	5.9	84.7
μ_t	0.0252	45.4	26.4	2.8	0.9	24.5
R_t^n	0.0008	22.3	16.7	2.4	52.7	5.9
π_t	0.0006	36.3	10.2	36.9	11.4	5.2

Figure 1: The Economy's Responses to a Tightening Monetary Policy Shock



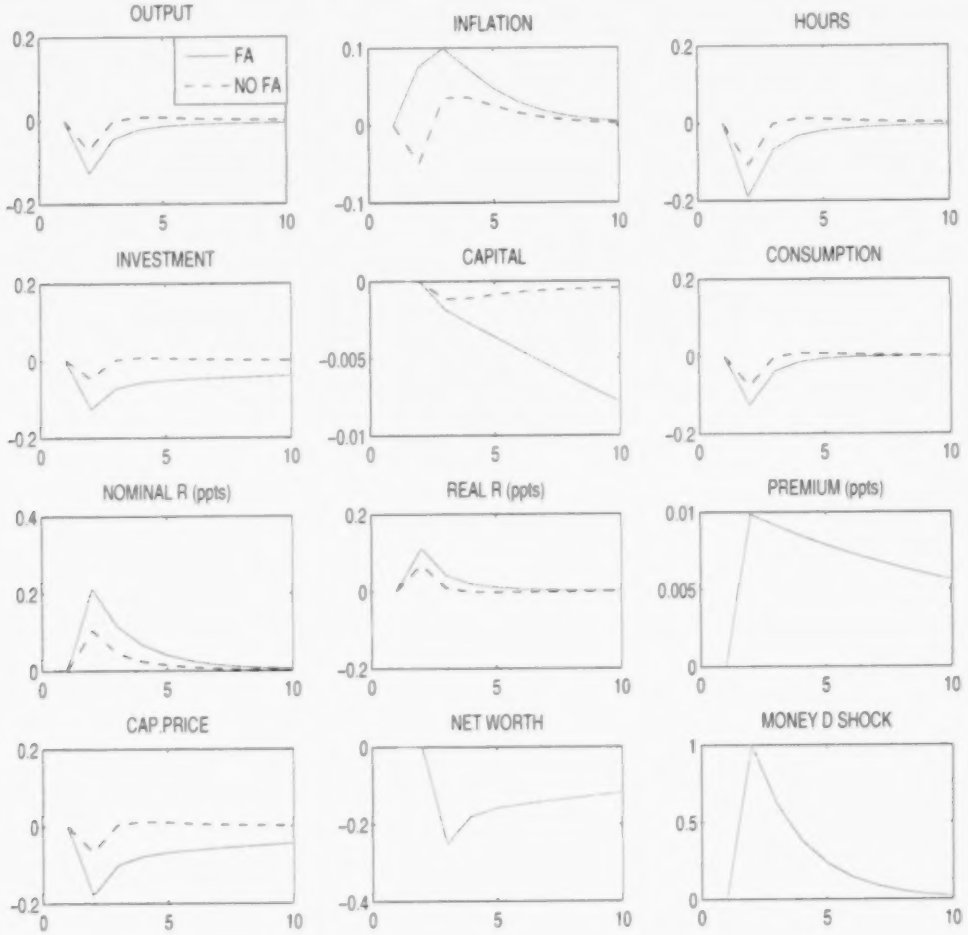
Notes: Responses from the FA model are shown by the solid red line and responses from the No-FA model are shown by the dashed blue line. The responses are percentage deviations of a variable from its steady-state values, except for variables that are rates (denoted by ppts). In this case, the lines represent the change in the rate in percentage points.

Figure 2: The Economy's Responses to a Positive Technology Shock



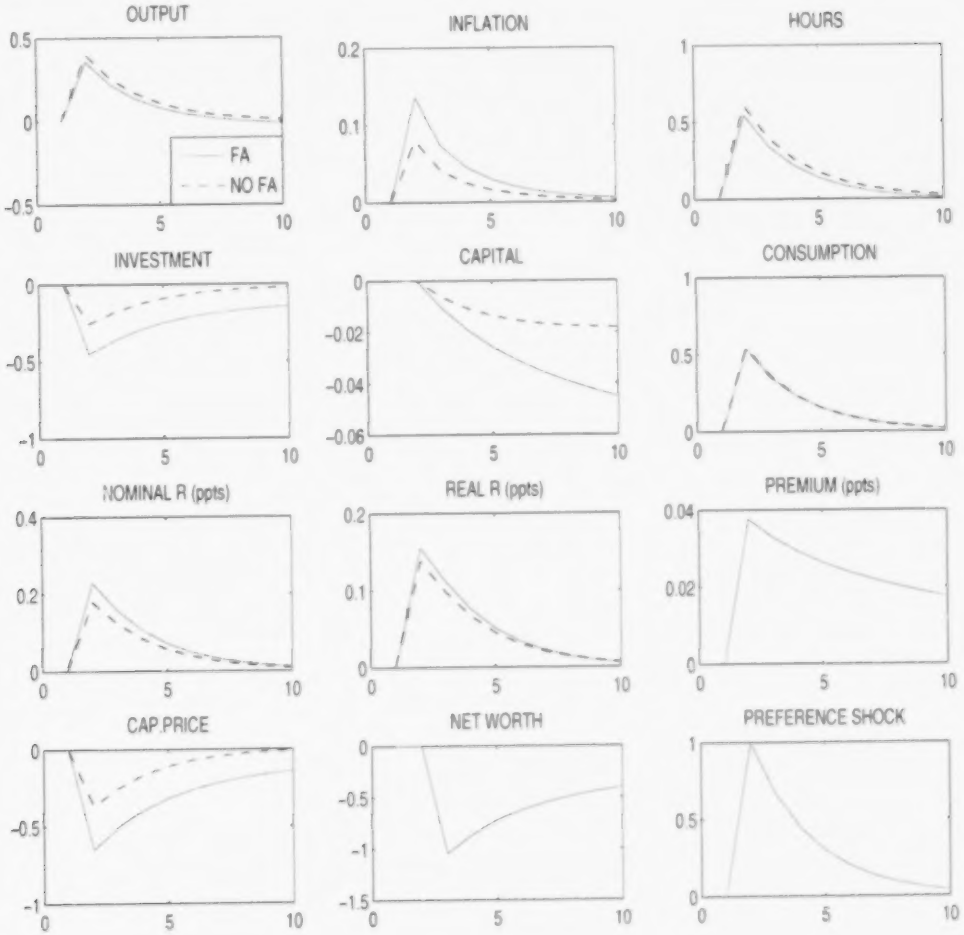
Notes: Responses from the FA model are shown by the solid red line and responses from the No-FA model are shown by the dashed blue line. The responses are percentage deviations of a variable from its steady-state values, except for variables that are rates (denoted by ppts). In this case, the lines represent the change in the rate in percentage points.

Figure 3: The Economy's Responses to a Positive Money-Demand Shock



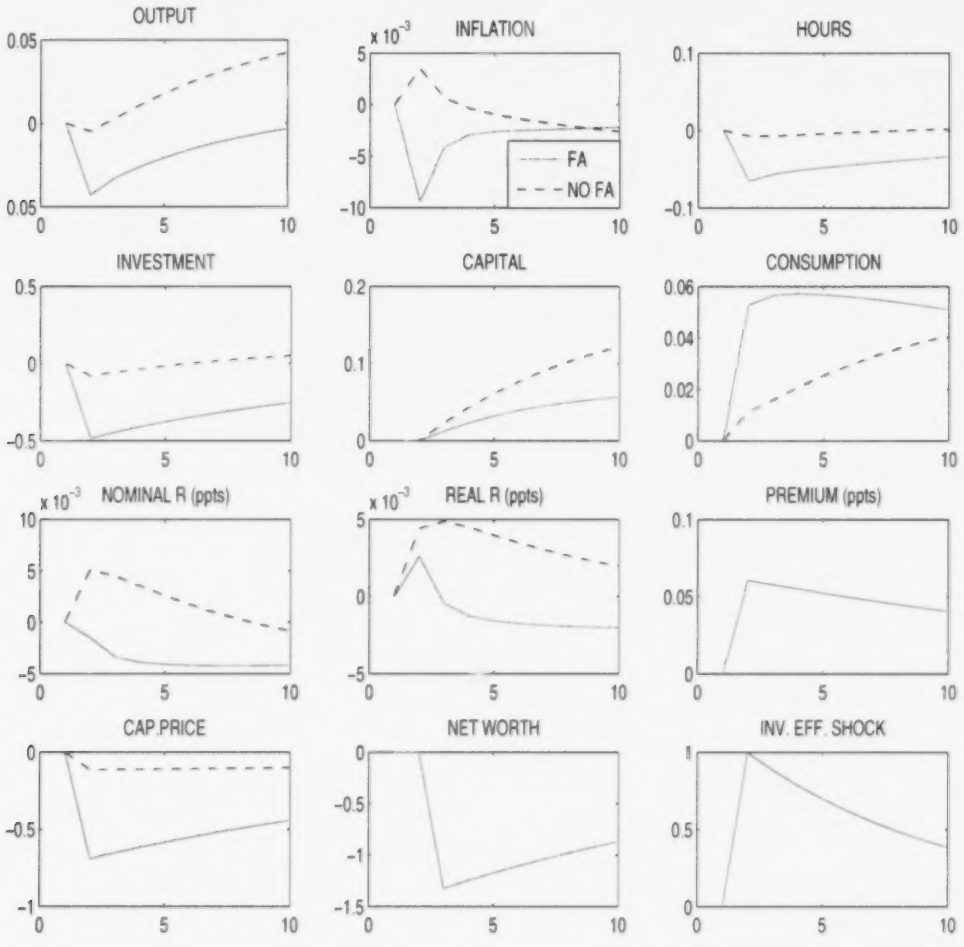
Notes: Responses from the FA model are shown by the solid red line and responses from the No-FA model are shown by the dashed blue line. The responses are percentage deviations of a variable from its steady-state values, except for variables that are rates (denoted by ppts). In this case, the lines represent the change in the rate in percentage points.

Figure 4: The Economy's Responses to a Positive Preference Shock



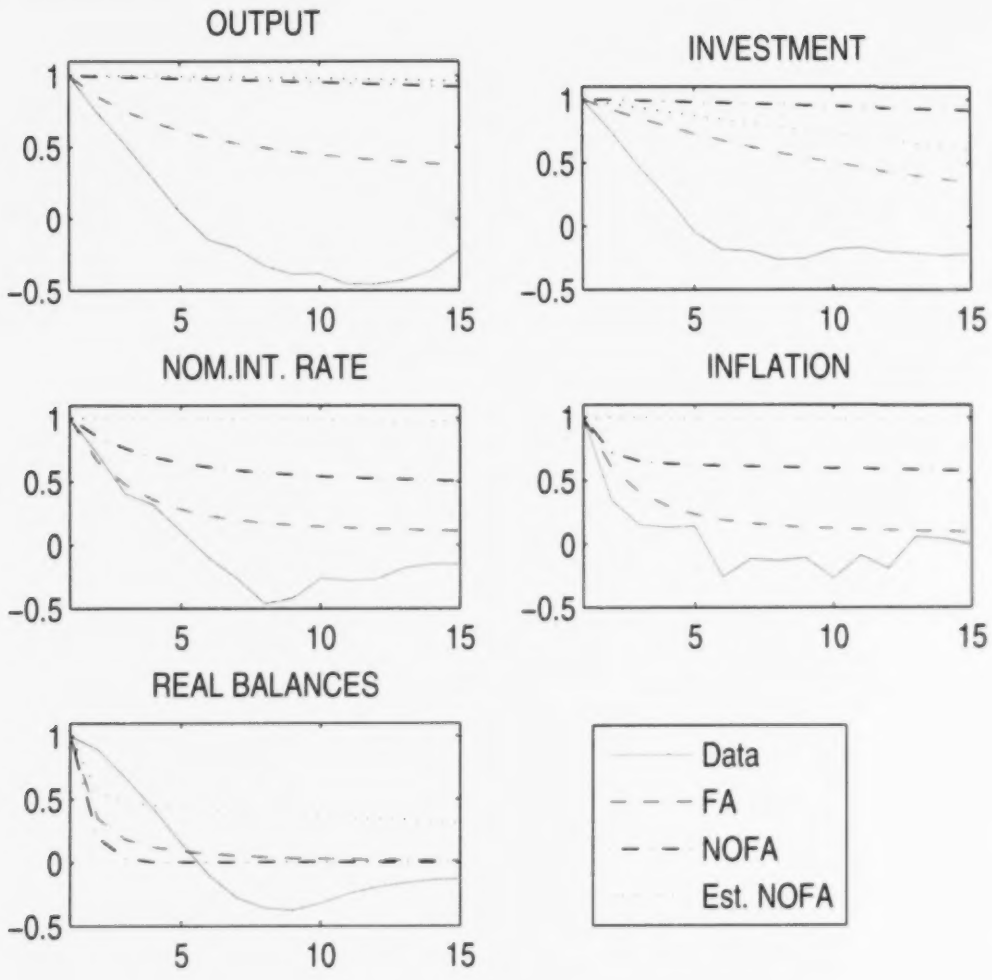
Notes: Responses from the FA model are shown by the solid red line and responses from the No-FA model are shown by the dashed blue line. The responses are percentage deviations of a variable from its steady-state values, except for variables that are rates (denoted by ppts). In this case, the lines represent the change in the rate in percentage points.

Figure 5: The Economy's Responses to a Positive Investment-Efficiency Shock



Notes: Responses from the FA model are shown by the solid red line and responses from the No-FA model are shown by the dashed blue line. The responses are percentage deviations of a variable from its steady-state values, except for variables that are rates (denoted by ppts). In this case, the lines represent the change in the rate in percentage points.

Figure 6: Autocorrelations



Notes: These plots show the autocorrelation functions for each variable implied by the three models considered (dashed red line, FA; dot-dashed blue line, No-FA; and dotted blue line, Estimated No-FA) and calculated using HP-filtered data in the 1979Q3 to 2004Q3 sample (green line).

Appendix A: The Non-Linear Equilibrium System

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \tag{A.1}$$

$$\left(\frac{b_t c_t}{m_t}\right)^{1/\gamma} = \frac{R_t^n - 1}{R_t^n}; \tag{A.2}$$

$$\frac{\eta}{1-h_t} = \lambda_t w_t; \tag{A.3}$$

$$\frac{1}{R_t^n} = \beta E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t}\right]; \tag{A.4}$$

$$R_t = E_t\left[\frac{R_t^n}{\pi_{t+1}}\right]; \tag{A.5}$$

$$r_{kt} = \alpha \frac{y_t}{k_t} mc_t; \tag{A.6}$$

$$w_t = (1-\alpha)\frac{y_t}{h_t} mc_t; \tag{A.7}$$

$$y_t = k_t^{\alpha}\left(A_t h_t\right)^{1-\alpha}; \tag{A.8}$$

$$y_t = c_t + i_t; \tag{A.9}$$

$$\bar{p}_t = \frac{\theta}{\theta-1}\frac{E_t \sum_{l=0}^{\infty}(\beta\phi)^l \lambda_{t+l} mc_{t+l} y_{t+l}/p_{t+l}}{E_t \sum_{l=0}^{\infty}(\beta\phi)^l \lambda_{t+l} y_{t+l}/p_{t+l}}; \tag{A.10}$$

$$p_t^{1-\theta} = \phi p_{t-1}^{1-\theta} + (1-\phi)\bar{p}_t^{1-\theta}; \tag{A.11}$$

$$E_t f_{t+1} = S\left(\frac{n_{t+1}}{q_t k_{t+1}}\right) E_t R_t; \tag{A.12}$$

$$E_t f_{t+1} = E_t\left[\frac{r_{kt+1} + (1-\delta)q_{t+1}}{q_t}\right]; \tag{A.13}$$

$$E_t n_{t+1} = \nu\left[f_t q_{t-1} k_t - E_{t-1} f_t (q_{t-1} k_t - n_t)\right] + (1-\nu) g_t; \tag{A.14}$$

$$k_{t+1} = x_t i_t + (1-\delta) k_t; \tag{A.15}$$

$$q_t = 1 + \chi\left(\frac{i_t}{k_t} - \delta\right); \tag{A.16}$$

$$\frac{R_t^n}{R^n} = \left(\frac{\pi_t}{\pi}\right)^{\varrho_\pi}\left(\frac{y_t}{y}\right)^{\varrho_y}\left(\frac{\mu_t}{\mu}\right)^{\varrho_\mu} \exp(\varepsilon_R); \tag{A.17}$$

$$\mu_t = m_t \pi_t / m_{t-1}. \tag{A.18}$$

Appendix B: The Steady-State Equilibrium

$$\mu = \pi = 1; \tag{B.1}$$

$$q = 1; \tag{B.2}$$

$$mc = \frac{\theta - 1}{\theta}; \tag{B.3}$$

$$R = R^n = 1/\beta; \tag{B.4}$$

$$f = r_k + 1 - \delta; \tag{B.5}$$

$$f = S(\cdot)R; \tag{B.6}$$

$$i = \delta k; \tag{B.7}$$

$$\lambda c = \left[1 + b\left(\frac{\pi}{\pi - \beta}\right)^{\gamma - 1}\right]^{-1}; \tag{B.8}$$

$$\lambda m = \lambda c b \left(\frac{\pi}{\pi - \beta}\right)^{\gamma}; \tag{B.9}$$

$$\frac{k}{y} = \frac{\alpha mc}{r_k}; \tag{B.10}$$

$$\frac{c}{y} = 1 - \delta\frac{k}{y} \tag{B.11}$$

$$wh\lambda = \frac{(1-\alpha)(\lambda c)mc}{c/y}; \tag{B.12}$$

$$h = \frac{wh\lambda}{\eta + wh\lambda}; \tag{B.13}$$

$$y = Ah\left(\frac{k}{y}\right)^{\alpha/(1-\alpha)}; \tag{B.14}$$

$$\frac{n}{k} = \frac{1}{1 - \gamma S(\cdot)R}\left(\frac{g}{k}\right). \tag{B.15}$$

Appendix C: The Log-Linearized Equilibrium

$$((1-\gamma)\lambda c-1)\,\hat{c}_t=\gamma\hat{\lambda}_t+\lambda\frac{(R^n-1)}{R^n}m\left(\hat{b}_t+(\gamma-1)\,\hat{m}_t\right)-\gamma\hat{z}_t; \quad \text{(C.1)}$$

$$\hat{b}_t+\hat{c}_t-\hat{m}_t=\gamma\frac{1}{(R^n-1)}\hat{R}^n{}_t; \quad \text{(C.2)}$$

$$h\hat{h}_t/(1-h)-\hat{w}_t=\hat{\lambda}_t; \quad \text{(C.3)}$$

$$\hat{y}_t=\hat{A}_t+\alpha\hat{k}_t+(1-\alpha)\hat{h}_t; \quad \text{(C.4)}$$

$$y\hat{y}_t=c\hat{c}_t+i\hat{i}_t; \quad \text{(C.5)}$$

$$\hat{w}_t=\hat{y}_t+\hat{mc}_t-\hat{h}_t; \quad \text{(C.6)}$$

$$\hat{r}_{kt}=\hat{y}_t+\hat{mc}_t-\hat{k}_t; \quad \text{(C.7)}$$

$$\hat{\mu}_t=\hat{m}_t-\hat{m}_{t-1}+\hat{\pi}_t \quad \text{(C.8)}$$

$$\hat{R}^n_t=\varrho_\pi\hat{\pi}_t+\varrho_\mu\hat{\mu}_t+\varrho_y\hat{y}_t+\varepsilon_{Rt}; \quad \text{(C.9)}$$

$$\hat{f}_t+\hat{q}_{t-1}=\frac{r_k}{f}\hat{r}_{kt}+\frac{1-\delta}{f}\hat{q}_t; \quad \text{(C.10)}$$

$$\hat{q}_t=\chi(\hat{i}_t-\hat{k}_t); \quad \text{(C.11)}$$

$$\beta\hat{\pi}_{t+1}=\hat{\pi}_t-\frac{(1-\beta\phi)(1-\phi)}{\phi}\hat{mc}_t; \quad \text{(C.12)}$$

$$\hat{\lambda}_{t+1}=\hat{\lambda}_t-\hat{R}_t; \quad \text{(C.13)}$$

$$\hat{\pi}_{t+1}=\hat{R}^n_t-\hat{R}_t; \quad \text{(C.14)}$$

$$\hat{k}_{t+1}=\delta\hat{i}_t+\delta\hat{x}_t+(1-\delta)\hat{k}_t; \quad \text{(C.15)}$$

$$\hat{f}_{t+1}+\psi\hat{n}_{t+1}-\psi\hat{k}_{t+1}=\hat{R}_t+\psi\hat{q}_t; \quad \text{(C.16)}$$

$$\frac{\hat{n}_{t+1}}{\nu f}=\frac{k}{n}\hat{f}_t-\left(\frac{k}{n}-1\right)\left(\hat{R}^n_{t-1}-\hat{\pi}_t\right)-\psi\left(\frac{k}{n}-1\right)(\hat{k}_t+\hat{q}_{t-1})$$
$$+\left(\psi(\frac{k}{n}-1)+1\right)\hat{n}_t. \quad \text{(C.17)}$$

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